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A STUDY OF THE SPECTRAL COMPLEMENTARIES.

I. INDIVIDUAL VARIATION OF THE NORMAL COLOR-SENSE.

II. SUBJECTIVE SATURATION OF SPECTRAL COLORS.

DISSERTATION

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Introduction.

The lack of a group of individual data pertaining to the wave-lengths of spectral complementary hues, which should be strictly comparable and should correspond to a white of definitely known spectral constituency has been deplored by recent writers in physiological optics. Comparative data on the luminosity ratios of complementaries is wholly lacking. In view of the ever-increasing importance which is being attached to the precise specification and measurement of color, accurate data pertaining to fundamental characteristics of the color-sense, such as complementary-hue data, is, a priore, almost certain eventually to be of value. The opportunity for a comparative study of the normal color sense afforded by such data has proved of unexpected interest.

The general mode of procedure which has always been followed in investigations of the spectral complementaries, has been to provide a photometric field one side of which can be uniformly illuminated by two superposed pencils of more or less nearly homogeneous light of arbitrary hue, the other side by a white of definite constant spectral character. There are then five variable conditions, namely, the representative wave-lengths and the intensities of the spectral components, and the intensity of the standard white; and two of them may be arbitrarily fixed at the start, the observer varying the other three until the two sides of the photometric field appear as near alike as possible. The finished observation may be completely described by recording three figures, to wit, the representative wavelengths of the spectral rays, and the ratio of their intensities.

The present experimental investigation of the spectral complimentaries differs from preceding investigations in the following particulars:

1) A group of individuals has been examined under identical experimental conditions.

2) In addition to complimentary wave-lengths, the luminosity ratios of complimentaries have been determined for some of the observers.

3) Important experimental details, such as size and absolute brightness of the photometric fields and character of the photometric junction, have been carefully regulated and specified.

4) The quality of the comparison white has been fully standardized.

The chief distinctive outcomes of this investigation have been 1) the derivation of numerical values for the relative subjective saturations of the spectral colors, based upon the luminosity ratios of complimentaries, and 2) a study of the individual differences of the normal color sense.

Apparatus:

The apparatus used is shown diagramatically in Fig. I. ABC is a Brace spectrophotometer. The 60° prism, P, of this well-known instrument is made up of two 30° parts cemented together with Canada balsam, one of the abutting faces normally carrying a narrow

horizontal strip of silvering. For the present purpose the strip of silvering was replaced by a half-silvered film covering the whole area of the abutting face. It will be readily seen that a mixture of the beams of light from the two collimators A and B is effected by the half-silvering. Perfect uniformity of the ocular field was gained by a chemically deposited film, after having freshly polished the glass surface upon which it was laid. Visible interference fringes, due to a slight difference in the refractive indices of the Canada balsam used to cement the halves together, and the glass of the prism, were eliminated by placing a fine, straight wire between the lower edges of the abutting faces, thus giving to the thickness of the balsam layer a relatively steep gradient without causing a doubling of spectral lines seen through the prism, which occurred if the wire was placed vertically.

The white comparison field is provided by a plane surface, E, coated with magnesium oxide, inclined at 45° to the axis of the ocular, C, illuminated from a direction perpendicular to the axis of C. The inner vertical edge of E intersects the axis of C. This edge is sharp, and capable of complete disappearance under conditions of photometric identity. The technique by which it is produced is due to Dr. A.H.Pfund. A piece of plate glass is "smoked" with magnesium oxide, a crack is started in it by a short scratch near one edge with a glass-cutter, and the glass is then broken by steady pressure. The break will generally run straight across, leaving a perfectly sharp edge beyond the glass-cutter scratch.

For the illumination of E, a hole was cut in the side of the otherwise light-tight prism-box. The illumination is accomplished by means of a small gas-filled tungsten lamp, of the automobile headlight type, having a short coiled V-shaped filament (L_3, fig.1). The lamp is mounted on a photometer car which runs on an optical bench in the direction CG. Immediately outside the prism-box, in the path of the light from L_3 to E, is placed a filter F, for the purpose of modifying the spectral distribution of the source L_3.

The filter is of a type recently developed by Dr. Pfund, and is capable of reproducing the spectral distribution of sunlight very nearly when used in connection with a suitable source. It consists of a liquid-cell having two distinct compartments, one of which contains an ammoniacal solution of copper sulphate, the other an aqueous solution of the sulphates of copper and cobalt. The proper concentrations of the different solutes give a smooth transmission curve which is almost the reciprocal or mirror-image of the energy distribution curve of the source, as may be seen by reference to Fig. 2. The spectral distribution of average noon sunlight at Washington, (Abbott)[1] was adopted as standard.

The energy distribution of the artificial source as compared to that of the adopted standard is shown in Fig. 2. The former was gotten by taking the products of the transmission of the filter, measured spectrophotometrically, and the energy distribution of the source L_3 determined from its color-temperature, a "Planck distribution" being assumed.

The color temperature of L_3 was maintained at approximately 3000°K. The constancy of the color-temperature of the source during the series of experiments was checked by means of another lamp of the same type which was preserved as a standard. The transmission of the filter was also checked and was found not to have changed at the completion of the experiments, three months after it was first sealed.

An actual comparison of the hue of the artificial source with that of noon sunlight at Baltimore in April, showed no color-difference as far as could be ascertained.*

Relative intensities of illumination on E are determined by the inverse square law, which was checked for this particular set-up and found to hold certainly within the required limits of accuracy. A rough estimate of the absolute intensity has been obtained from the rated candle-power of L_3.

The diameter of the ocular field is restricted by a diaphragm with circular aperture, D, Fig. 1, placed immediately in front of the ocular lens, for which purpose three interchangeable diaphragms are provided, having apertures subtending reppectively at the ocular slit angles of 1°, $1\ 1/2^\circ$ and 2°. The $1\ 1/2^\circ$ field was used in all the final observations. The ocular slit is fixed in dimensions, being 3.2 mm. by 0.43 mm wide.

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* N.B. Data on the color temperatures of sunlight recently obtained by Priest[2] show that the color temperature of average noon sunlight at Washington is approximately 5000°K, thus this value may be taken as approximately specifying the standard white here used.

The spectral sources L_1 and L_2 are 50-watt gas-filled tungsten lamps with milk-glass bulbs, bearing the trade-name "Colonial Mazda". These lamps, together with L_3, were operated on the same 120 volt storage-battery circuit, so that relative variations in intensity were negligible.

The intensities of the spectral stimuli are adjusted by varying the widths of the collimator slits. Wave-length adjustments are accomplished by rotation about the axis of the prism of collimator B and the ocular, C, both being provided with tangent screws. Wave-length determinations are made by means of an angular scale with a vernier associated with C. The smallest division of the vernier is 1/100 degree, amounting to something less than 1μμ on an average throughout the spectrum. Since B is moveable and has no scale, it is necessary to provide in the spectrum of B a fixed line to serve as a fiducial mark for wave-length measurements: conveniently accomplished by introducing before the slit of B an absorption cell containing a solution of neodymium chloride. An accessory eyepiece is used for setting on the absorption band, and for calibrating the instrument from known line spectra.

The ideal type of spectral stimuli would be those whose entire range of wavelengths lie within the threshold of hue difference discrimination. Such stimuli, although not necessarily objectively homogeneous, could be appropriately called "monochromatic". With the present apparatus, however, there were practical difficulties in the way of realizing monochromatic stimuli, and it seemed simpler to correct for possible errors introduced by the use of wider spectral

bands. Errors of two types might thus arise:

(1) The dominant hue of the stimulus composed of the wide band may not coincide with that of the monochromatic stimulus having the same mean wavelength.

(2) It is possible that the subjective saturation of the composite stimulus may suffer a diminution, thus affecting the complimentary luminosity ratio. It is interesting to notice in the latter connection that the shape of the spectral "color-triangle" indicates a marked variation throughout the spectrum of the minimum objective purity necessary for undiminished subjective saturation, with a maximum in the blue-green region at or near the "peak" of the triangle. In the spectral stimuli used for the final observations a special investigation failed to show either a diminution in saturation or shift in hue as compared to sensibly monochromatic stimuli of the same mean wavelength. The average widths in wavelengths increments of the spectral bands constituting the stimuli are stated in the results.

The luminosities of the colored stimuli were determined in the larger part of the observations by a modification of the direct equality-of-brightness method which has described by Pfund.(3) Smoke-glass wedges, the density gradients of which are vertical and are opposed in direction, are placed in the spectral and standard white beams (wedges not shown in Fig. 1). The photometer field then has the appearance shown in Fig. Ia. The observer's criterion for equality of luminosity is that of equal contrast on the upper and lower halves of the field, or of minimum distinctness of the dividing line at the mid-point.

The introduction of the wedges appears to reduce very materially the difficulty of making a "heterochromatic" brightness comparison to an observer who has had no previous training. Pfund has found that the luminosity values so obtained tally closely with the flicker-photometer. In taking observations, the observer is permitted to color-match the two halves of the field, and to make luminosity determinations on the neutral mixture without use of the wedges, which are put in place only for the photometry of the separate colored components. The wedges are mounted in such a manner as to fit accurately into place, and, as proved necessary, the spectral wedge is accurately compensated with a wedge of clear glass to avoid deviation of the spectral beam when it is inserted.

Since the glass of which the wedges is composed is not perfectly non-selective its spectral transmission data had to be ascertained, after which the wedges were calibrated in place. The luminosity without wedges, L , of a spectral stimulus of mean wavelength λ could then be obtained in terms of the luminosity with wedges, L', by the formula $L = \frac{L' T}{T_\lambda}$ where T is the factor by which the luminosity of the standard white is reduced by the introduction of wedge, and T_λ is the transmission of the spectral wedge at wave-length λ.

Procedure:

A total of seven observers has been examined for complementary wave-lengths, of whom five furnished luminosity data. Of the latter, two had had much previous experience in color match-

ing, and one some experience in spectral photometry. One observer (the writer) furnished a complete set of data. The remaining observers furnished observations for only four pairs of complementaries, selected at about equal intervals on the color-triangle of König. One color of each pair was given in advance, the observer adjusting the wave-length and intensity of the other spectral stimulus, and the intensity of the standard white, so as to form an exact match. The intensity of the given stimulus was selected in such a manner as to demand an intensity of the neutral stimulus approximating to a certain predetermined value, of the order of 75 to 100 photons[4] on the retina of the observer's eye.

Two of the chosen pairs of complementaries involved the terminal hues of the spectrum. The extreme red beginning about the line C, has been shown by Abney to be practically constant in hue, which same is indicated by L.A. Jones' "scale of equal hue differences" for the spectrum.[5] Accordingly the stimulus representing the extreme red was permitted to include nearly the entire region from line C to the end of the visible. At the other end of the spectrum, Jones' work shows continual alteration of hue out to about λ 400 μμ. An investigation by means of complementaries does not appear to verify this result. A very careful set of observations shows that all the violet wave-lengths beginning at about λ 445 μμ have exactly the same complement, hence cannot vary in hue. The average deviation, Δλ, of observations on the complement of this region is probably least of similar quantities for any part of the spectrum (unfortunately the exact value cannot be stated because it lies with-

in the wave-length value of the smallest division of the wave-length scale of the spectrophotometer, about 1 μμ in this region of the spectrum), so that the fact is very good evidence that no actual hue change is present in the above-defined region. It is perhaps possible that the eyes of different individuals may vary in this respect, but evidence to be adduced hereto indicates the contrary.

The apparent hue change in this region shown by the results of Jones' investigation is probably not actual, but is occasioned by the increasing aberrations of the optical system of the eye. It was observed that although possessing the same complement, two stimuli composed respectively of wave-lengths centering about λ. 445 and λ. 400 could be differentiated by their appearance. At first a different in subjective saturation unaccompanied by change in hue was suspected, but by a determination of the complementary luminosity ratios of the two stimuli, which gave identical results for each, it was found not to occur. The apparent distinctness of the visual impressions received is evidently wholly due to a very marked increase in chromatic aberration. Probably the total discriminability due to this cause in the extreme violet, is sufficient to account for several steps of just perceptible difference. Any one familiar with the spectrum of the mercury arc is likely to question the inference that no hue-difference exists between the so-called "blue" line (λ. 436) and the "violet" lines (λ.405). No doubt an actual hue difference does here exist, its explanation, however, lies in the fact that these lines are of very inequal effectiveness in exciting the visual processes, as they fall upon the retina in the image of

the arc spectrum. The extension of the blue into the violet as a result of increased intensity has been noted by many investigators.

Mr. A. Ames , Jr. as a result of his extensive study of the dioptrics of the eye has previously suggested that chromatic aberrration may be an aid to color-blind persons in distinguishing between certain colors or colored objects and that this circumstance is perhaps a reason for the success of the Holmgren yarn test-object, by virtue of its lack of sharp outlines. There can be little doubt but that the effect is present in the extreme violet, in normal vision. The region from about λ 445 on, therefore, was employed for the extreme violet stimulus. A blue filter was placed over the violet slit to eliminate stray light. The remaining selected stimuli were an orange λ 609. 5, and a yellow λ 581.

A color-match having been obtained to the observer's satisfaction, he then proceeded to determine the luminositi sof the neutral mixture and each of its components separately, in the manner outlined, several photometric settings for each luminosity being made without disturbing the adjustments pertaining to the colored stimuli. The luminosity of the mixture was ascertained as a check on the heterochromatic luminosities which should presumeably summate to the same value.

Results:

The numerical results are comprised in Tables I, II and III. Table I contains the results of the detailed investigation of one of the observers. The first two columns give the mean wave-lengths of the complementary stimuli, the third column the luminosity ratio of the longer to the shorter wave-length, the fourth column the difference of the sum of the separate luminosities of the colored stimuli and the luminosity of the white mixture, expressed in percent. All luminosity determinations were made by the simple equality-of-brightness method without the use of wedges. The figures in the fourth column are in a general way a gauge of the accuracy of the luminosity measurements. The fact that the former are nearly always positive indicates that the effect of hue in the colored stimuli has been to increase apparent luminosity. Mention should be made of the fact that the comparison white of this set of data differs very slightly in quality from that for which the spectral distribution has been given, which latter was adopted for all other observations except as noted.

Table II contains all the wave-length data apart from that given in Table I. It will be noticed that a selected spectral stimulus other than the four described above, was used upon one of the observers, the reason being that one of the standard stimuli was to him without a spectral complement. Nearly all of the observed wave-lengths are the averages of two or three settings. In many cases the deviations of the individual settings were less than the smallest division of the wave-length scale. ($<1 m\mu$). In the last three sets of observations recorded in the table, the quality of the

comparison white was altered in divers ways, as indicated. Under the headings $\Delta\lambda_1$ and $\Delta\lambda_2$ are given the widths in millimicrons of the wave-length bands comprised in the spectral stimuli corresponding to λ_1 and λ_2. The observer who furnished the data of Table I repeated observations both of wave-length and luminosity under the standardized conditions of Table II so as to obtain data comparable with that of the remaining observers, which data is included in Tables II and III.

In Table III are given the luminosity ratios of longer to shorter complementary wave-lengths for five observers whose wave-length data appears in the preceding table. The given spectral stimulus of the complementary pair for which the different ratios apply is indicated at the top of each column. The figures in the last column are analogous to those of the column similarly headed in Table I, in the present case, however, being arithmetical averages for all the observations of a stated observer. For purposes of comparison, data given by Helmholtz[6] are included in Table III, which, apparently, are the only previously data extant of this sort obtained by direct observation. Although in the table they are placed opposite the wave-length labels which apply to the rest of the data, Helmholtz designated the complementary pairs for which they hold merely by color names. He found that the values of the ratios were subject to considerable variation with change in the level of absolute luminosity at which observations were made, and gives two sets of figures corresponding to "low" and "high" absolute

luminosity, of which the latter are here reproduced.

The luminosity ratios of complementaries may be obtained indirectly from "primary color"data, if taken in conjunction with the "visibility function" of the average eye (the personal visibility data supplied by König and Abney without much question depart farther from the fact than do the average data.)

It is known that wide variations occur in individual visibility characteristics, thus little is to be gained by a comparison of the results afforded by the individual color-triangles.

The calculation has been carried out on the basis of the convenient average color triangle reduced from the data of Abney, König and Dieterici by Mr. E. P. Weaver, as published in the 1920-21 Report of the Committee on Colorimetry of the Optical Society of America,(1) together with other necessary data all taken from the same Report. The results are given in Table III. Too much reliance should not be placed on the latter, however. The agreement between the color-data and the average visibility function (of tables 6 and 2 of the above Report) is not as good as it should be, for if a different set of three points is employed to fix the luminosity coordinates of the color-plane, materially different values for the ratios may result. The points used in the calculation were: $\lambda\lambda$ 450, 550, 660 $\mu\mu$. Thus if λ 450 is taken instead of λ450 the violet ratio is above 100. In addition to these values obtained from the average color-triangle, the writer has also calculated values

from the the individual color-trianlge of König. It was found that if the above wave-lengths were used (with average visibility data for sunlight) to deter ine the color-plane, König's violet elementary had to assume a definitely negative intrinsic luminosity, but if 1.490 were used instead of 1.450, all the elementaries had positive luminosities and the ratio for the extreme violet came out about 6. The fact that this value departs so widely from that for the mean color-trianlge makes the probable error of the latter extremely large.

It may also be mentioned that v. Frey and v. Kries,(8) as well as Angier and Trendelenburg(9) recorded slit-wieths giving complementary amounts of spectral complementaries, with other data pertaining to the calculation of complementary energy-amounts, which latter together with the average visibility function would be capable of yielding luminosity ratios, but in both cases there have been omissions of certain important data such as to leave an element of uncertainty and since also the results would be without value for individual comparison for the reason stated above, the reduction of these data was not undertaken.

Discussion of Results.

I. Derivation of numerical values for the subjective saturation of spectral colors.

Helmholtz,(10) who was the first to notice that complementary amounts of the spectral complementaries in general differ in luminosity, attributed the fact to a difference in subjective saturation among the spectral colors. From the values which he estimated

for these luminosity ratios, together with observations of similar character upon colored mixtures involving the central green, he was able to arrange the spectral colors in order according to their degree of subjective saturation, rating violet as the most saturated color, and yellow the least. He did not, however, attempt a numerical evaluation of the relative saturations, stating that he was constrained from so doing, by the very considerable extent to which the values of the luminosity ratios of complementaries were subject to the influence of the Purkinje phenomenon, at the level of absolute luminosity at which his experiments were carried out.

Very recently Troland[1] has investigated the quantitative relations in which the spectral colors stand in regard to saturation, in a manner quite different. Troland noted that the spectral colors, if alternated with white in the flicker photometer, have different critical flicker frequencies. Assuming that the critical flicker time of a color under these circumstances is in a way a measure of the extent to which it differs from white, and hence that colors equally saturated in the subjective sense, ought to have equal flicker-times, he added white to each of a set of representative spectral colors, in such amount as to give to each diluted color the same flicker-time, under which conditions the amounts of white which had to be added to the various colors served as a basis for estimating their relative saturations in the pure state. The outcome of the investigation is in general agreement with Helmholtz's conclusions, indicating that the maximum saturation is to be found at the violet end of the spectrum, and that a minimum of saturation occurs in the yellow. The position of the minimum is fairly definitely localized at λ 575 $\mu\mu$.

Use can be made of this information to correlate the present data for the luminosity ratios of complementaries in terms of the subjective saturations of the spectral colors, if it be postulated that the subjective saturations of two complementaries are inversely proportional to their luminosities. It happens that the complements of practically the entire blue and violet regions of the spectrum, are crowded into the region of rapid hue-changes between λ570 and λ 580. Now since the saturation function has a minimum at about the middle of this interval, and does not increase rapidly on either hand, it may be assumed without overstepping the limit of accuracy to which this investigation lays claim, that the saturation is constant throughout this interval, and one may conveniently assign it a value of unity, whereupon the saturations of all the colors whose compliments lie in this region - become simply the reciprocals of the luminosity ratios of the several complementary pairs, and may be plotted.

Figure 3 exhibits the data of Table I so plotted. Complementary pairs one member of which lies in the interval of λ570 to λ580 are represented by the crosses, and trace the saturation curve from the extreme violet to λ482.5, which is the compliment of λ580. Here another region of rapid hue variation commences, and all the complements of the remaining hues at the red end of the spectrum, from λ 580 to the extremity, are contained in the brief interval from λ482.5 to λ496.5, which is the compliment of the extreme red. A short extrapolation of the curve in the blue-green is thus sufficient to fix values for the saturations of all the hues beyond λ580.

Enough points have been taken in the blue to determine the exact course which the curve follows as it approaches the region to be extrapolated, so that the extrapolation could be made with maximum assurance. In Fig. 3 the extrapolation has been carried out and corresponding points lying in the extrapolated region of the curve, and in the region complementary to it, are marked by circles.

In the gap between λ496.5 and λ570 a point has been tentatively plotted, from considerations of a similar nature to the foregoing. A cell containing a concentrated solution of $NaCrO_3$ was placed in the standard white beam of the color-matching apparatus, thus producing an intense yellow on that side of the photometric field. This yellow proved to be nearly as saturated as the spectral yellow, and its dominant hue could be readily determined by comparison with a spectral stimulus and was λ580. A spectral green and orange of λ534 and 613 then were selected, on the principle that they lie at equal distances on either side of λ580, not on the wave-length scale, but on the scale of equal hue-differences of L.A.Jones. A mixture of these two stimuli was caused to match the chromate yellow, and the luminosity ratio of orange to green determined and found to be 0.8.

The point λ613 lies on a portion of the curve already determined, so that the corresponding point can be plotted. In Fig. 3, the two points are marked by x's. The portion of the curve filling in the gap in the green and passing through the point at λ534 is drawn dotted, to indicate its highly tentstive nature. One notices

that the saturation at 1. 534 turns out somewhat less than at 1.613, which Troland's results also show.

It is evident that a very rapid change in direction of the saturation curve must occur at some place in the neighborhood of the blue-green, which introduces considerable uncertainty into the process of extrapolation in this region, even though the distance to be extrapolated is small. It is even possible that a second minimum of saturation may occur at a point not remote from the blue. Helmholtz in his treatise on the extension of the Weber-Fechner principle to apply to the elementary col_or-variables,(12) the very great interest of which has perhaps escaped full recognition, has plotted the spectral colors in what is probably nearly their correct relations to the elementary variables. Thus plotted the color diagram has the unique property that, near the center of the diagram a just perceptible hue difference is represented by the same distance at every point and in all directions. Thus the distance between the locus of any color and the white point at the center, if not too great, represents the number of just perceptible hue-steps between the color and white, in other words, represents its subjective saturation from a new standpoint. A glance at Helmholtz's diagram, based on the data of König, shows that the locus of the spectral colors, which falls near enough the center for the above relation to hold approximately, approaches the center in two places, one in the yellow and the other in the blue, between

which there is a distinct departure toward the peripheries of the triangle showing a point of maximum of saturation at λ520. However, König's data for the blue end of the spectrum are probably considerably in error. Were Helmholtz's transformation based on accurate data, the rapid rate of increase of complementary luminosity ratios in the blue suggests that an actual second minimum of saturation would not be found but merely a sharp bend in the curve in the green, which would correspond to the point of inflection drawn into the curve of Fig. 3.

Having determined the form of the saturation curve based on the data of one observer, it may be readily modified to suit the average luminosity data of several observers contained in Table III. In this list of observers, unfortunately one (AE) proved to be anomalous in a singular way, hence his data cannot be accorded the same weight with the rest. No marked change will appear in the curve, except in the violet, where it will rise only to about the height shown by the dotted line, Fig. 3.

Comparing the present results with those of Troland, it is seen that the latter fail to reveal the multifold increase in saturation in the short wave-lengths, although agreeing in their general aspect with the above. In view of the very different way in which saturation has been defined in the two cases the fact that different numerical results have been obtained is not as significant as the

the agreement in general character among these results as well as those from Helmholtz's color-diagram which involves still another definition of saturation.

It is possible that under different conditions a still greater value for the complementary luminosity ratio of the extreme violet would be obtained, more nearly agreeing with that afforded by the average color-trangle. Helmholtz found that the violet ratio was most altered by the presence of the Purkinje phenomenon, the complete absence of which in the present investigation is not assured. Ives[13] sets as a safe upper limit to the range of the effect a retinal intensity of 75 photons for a luminous field confined to the fovea centralis. Troland[4] finds for himself as an observer the limits can be carried much lower, but suggests that individual differences in this respect may exist. In the present investigation the neutral comparison stimulus has an intensity above 75 photons, but the intensities of the spectral stimuli were necessarily lower, especially in the case of the violet, the photometry of which must have taken place at a level of three or four photons. The saturation obtained for the violet must therefore be looked upon as a lower limiting value which may be even doubled in the complete absence of the Purkinje effect. In fact, it was noticed that at if in the process of finding the violet ratio, instead of taking the directly observed luminosity of the violet stimulus, a value for the latter is obtained indirectly by subtracting the luminosity of the complementary stimulus from that of the mixture, the ratio

becomes as high as 90 or 100, indicating that the fraction of luminosity due to the violet component of the mixtures is nearly if not quite subliminial. This value, on the other hand, is probably too high, since the effect of hue in the yellow complementary stimulus would cause its luminosity to be overestimated with the result that the difference value for the violet luminosity would be too small, notwithstanding the fact that heterochromatic luminosity comparisons may be made with greatest precision at just this point in the spectrum. The latter figure is thus an upper limiting value.

The essential fact in regard to the extreme saturation, in the present sense, of the violet has not been unrecognized. Abney has pointed out its significance in relation to the luminosity coefficients which must be attached to the "fundamental color sensations" of his selection. Others have done similarly for the "sensation" data of König.

Discussion of Results.

2) Individual variations of the color-sense.

Even among individuals whose perception of color is in no sense defective, variations in color-characteristics are known to be considerable. Such variations are evinced by individual differences in color-match equations. Maxwell was first to suggest as a cause a variation in amount of the yellow pigment present on the "macula lutea" of the retina. Helmholtz considered that individual differences in color-characteristics are too great to be wholly traceable to this source. (8) Kries and Frey in the report of their joint investigations of spectral complementaries have shown that the discrepancies in their individual data are of a sort which a variation in the macular pigment would lead them to expect. In the discussion, they have plotted on the color-triangle the loci of complementary pairs and have indicated that interposition of a yellow tinted medium in the optical system of one of them would induce variations qualitatively of the right sort to cause the two sets of data to coincide; moreover, that the existing variations are superficially similar in character to those found between the macular and extra-macular vision of a given observer. Hering (15) compared fresh mounted specimens of the human retina and verified the occurrence of marked differences in pigmentation among retinas from separate individuals. He also allowed

"red-sensitive" persons to set up a color-match between a mixture of yellow and blue-green and a monochromatic intermediate green, and found the match held also for "blue sensitive" persons when the latter viewed it through mounted retinas of the proper pigment densities. Hering entertained the opinion that variations due to pigmentation of the macula form a very considerable part, but not the entirity, of the total variability of the normal color-sense.

Complementary wave-length data afford a means of determining definitely the part played by the macular pigment in producing individual variations. Unfortunately previously extant data of this sort are ill-suited to comparison, because, as is well-known, the standard whites previously used have not been definitely enough specified as to quality, and have evidently varied considerably in separate investigations, while in no instance were more than two observers examined under uniform conditions. In the present investigation, as has been stated, seven observers were examined under identical experimental conditions. Individual differences hence ought to rest solely upon actual differences in the color-characteristics of the individual. The character of the individual differences can conveniently be exhibited by plotting the data on trilinear color-coordinates.

It is a fundamental property of the color-trianlge that the straight line joining any two given colors on the diagram is the

locus of all hues produceable by their mixture. Accordingly, if on the color-trianlge the loci of the spectral hues be known for a particular observer, and his complementary wave-lengths be joined by straight lines, all the lines will have the white-point of the diagram in common, and therefore all will intersect in a point. Now, if a second observer's complementary wave-lengths were plotted on the first observer's spectral locus, the lines joining corresponding pairs would not in general contain the white-point of the first observer. But if the sole cause of difference between their color-characteristics were variation in the pigmentation of the optical media of the eye, the second observer's several lines would still intersect in a point, because the selectivity of the optical media would be effective only in altering the relative intensities of the spectral stimuli without altering their hues, while altering the hue of the white stimulus. The white-point of the second observer thus would be displaced from that of the first in the direction in which the quality of the standard white is modified by the addition or subtraction of the dominant hue of the pigment present in varying amount so that a straight line drawn through both points will cut the spectral locus at the dominant hue of the pigment, or its complement. Consequently, if one plots the data of several observers whose color-characteristics vary only from this cause all the white-points will be found to lie on a straight line. In the presence of any other kind

of individual variations, the complementary data of one observer plotted on the spectral locus of a second, would not in general determine lines having a point in common.

The data of table II of the present investigation have been plotted on the color-triangle of König. For this purpose, the color triangle data contained in table 26 (column K) of König's treatise entitled "Die Grundempfindungen in normalen und anomalen Farbensystemen" , (p. 316, Gesamm. Abhandl.) were plotted on a large sheet of coordinate paper. Points at 480, 490 and 500mμ were interpolated from König's "Grundempfindungen" curves. As a further aid to interpolation in plotting the complementary data, the distances between the plotted points were divided into a number of equal parts corresponding in each instance to the wavelength increment in millimicrons between the points.

Fig. 4 is a photographic reduction of the above diagram, on which have been plotted not only the data of the present investigation but also all other available complementary-wavelength data meriting consideration.* If all the lines joining the complementary pairs had been drawn in full, the diagram would be utterly confused. It is hoped that by tracing merely the polygon or other figure which each set of lines forms about the point the sum of whose distances from all the lines is least, and by using different styles of delineation, the reader will be enabled to differentiate the separate groups. The heavily inked configurations belong to the observers

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* The only exceptions are the data of R. Schelske (16) and those of König's "anomalous" observers Zehnder and Becker (17), which are omitted because of their paucity.

of the present investigation and may be associated with the corresponding data in Table II by the attached initials. The authorship of the remaining data is indicated in the legend accompanying the figure.

It may be seen at a glance that variations in the pigmentation of the ocular transparent media are responsible for far the greatest share of the variability depicted, and that the variation of the white point is very considerable. It is not, however, as great as might be inferred prima facie from the distances between the points on Fig. 4, because as is evident from the length of the wave-length divisions along the margin of the triangle the diagram is distended in a direction parallel to that along which the major variation takes place. Helmholtz's transformed coordinates would give better visual representation of the amount of hue variation corresponding to the variability of the white-point, but it was for the reason that the individual points are further separated on König's diagram that the latter was adhered to. On casual inspection it is also evident that the white-point variation is due in the main to the addition or subtraction of one specific hue. A detailed study reveals that the white-points i.e. the points whose summated distances from all the lines of a set, are least, of five of the observers of the present investigation lie sensibly on a straight line, the line ab in the diagram. Observer (P.F.G.) whose white-point in the figure departs perceptibly from the line ab made wave-length settings with distinctly lower

precision than any of the other observers. The amount by which his white-point departs from the line of the others is certainly within the limit of observational error. His data are of interest chiefly in that they show no exception to the general conclusions which can be drawn from the investigation.

Observer (AE) is distinctly anomalous. The line ab cuts the spectral locus at λ 572.5 mμ, which may therefore be regarded as the dominant hue of the macular pigment, taken in conjunction with the spectral distribution adopted as standard in these investigations. As an interesting test of the correctness of the considerations upon which the preceeding argument is based, an attempt was made to modify the conditions of the experiment in a manner capable of imitating for a given observer the state of color-perception which would exist as a result of variation of the transmissive properties of his ocular media, of the sort which has been postulated. A thin cell containing a rather dilute solution of sodium chromate was found experimentally to possess in conjunction with the standard white illuminant a dominant hue very nearly that given by theory for the macular pigment. With the yellow filter placed in the standard white beam the writer obtained a set of complimentary data, included in Table II under an appropriate label, which is shown plotted in Fig. 4 at S'. The result is, of course, what the theory of the color-triangle predicts. The hue of the chromate solution is evidently very slightly toward

red as compared to the macular yellow.

Two other sets of observations were also made by the writer, in which the color-temperature of the standard white was varied along approximately the black-body route. The primary object of the investigation was to discover how nearly the effects of such variation imitated the individual variations due to the macular pigment. One color-temperature, somewhat higher than the adopted standard , was gained by increasing the filament-current of the lamp which served as the source. The second abnormal color-temperature was obtained by removing the "sun-light" filter from the path of the light from the source, which was then operated at its normal filament-current corresponding to a c. t. of 3000°K. Both sets of data are included in table II under suitable captions. The point determined by the first set of data falls so near that for the normal colortemperature of the same observer that it has not been plotted in Fig. 4. The second point is shown at S". The line passing through this point and that for the normal color-temperature labelled (RHS) cuts the periphery of the diagram at c, corresponding to a hue-change in the standard white imitable by addition of λ581.5. Line c is of course the secant of the locus of the black-body series of hues, the tangent of which at the point where it intersects the "white" line would cut the spectral locus a trifle nearer a, but not near enough to be able to compensate for pigment-variations by mere changes in color-temperature of the standard white. The white-points of observers not examined in the present

investigation obviously cannot be expected to lie exactly along the line ab on account of non-conformity of the various standard whites. It is to be noticed, however, that they do without exception lie very close to the line, their mutual displacements in a direction parallel to the line being of a higher order of magnitude than the distances by which they depart from it. In every case some species of daylight was used as the standard white illuminant. The variations in the standard whites are probably greater than the departures from the line might seem to show, however, since the larger part of the variation would likely occur in the direction from yellow to blue, hence, parallel to the line.

It may be noticed, however, in cases of investigators who have worked in pairs, that the white-points lie on lines nearly parallel to ab. Angier and Trendelenberg have points so nearly coincident that conclusions can scarcely be drawn but the fact is well brought out by the data of von Kries and von Frey, and König and Dieterici. The latter pair present a particularly interesting case. Not to add to the complexity of Fig. 4 by drawing another line, the data of the last-named individuals has been plotted separately in Fig. 5 in a different fashion, which illustrates very closely the point in question. Fig. 5 is a copy of "Fig. 4" of König's paper "Die Grundempfindungen in normalen and anomalen Farben systems" (on page 263, Gesamm. Abhandl.) the data for which is contained in Tables VIII and IX of the same treatise. The solid curves

marked K and D represent the complementary wave-lengths of König and Dieterici with sunlight as the standard white illuminant, with longer wave-lengths as abscissae and shorter ditto as ordinates. They will be observed to intersect at about λ. 578-477. The third solid curve marked K' is added to the original diagram from certain data given out by König several years later in a paper entitled "Quantitative Bestimmungen an complementaren Spectralfarben" (Table of results on page 375, Gesamm. Abhandl.), which data specify the author's complementaries at a point on the retina about 3° below the normal fixation point, hence in extra-macular territory. A secondary source was here used for the standard white, but was very carefully matched to sunlight according to the author's statement. Apparently the later investigation was entirely independent of the first. König does not make use of the wave-length data gotten in the later investigation.

A glance at Fig. 5 of the present paper shows that curve K' intersects curves K and D almost precisely at their common point. This is of course equivalent to the circumstance that the "white-points" on the color triangle corresponding to the three sets of data should lie on a line.

In Fig. 4 of the present paper the point K' corresponds to curve K' of Fig. 5. It may readily be seen that the three points considered do lie on a line altho the line has not been drawn. The principal differences which exist between the color-vision of these two individuals are therefore of exactly the same type that exist

between the macular and extra-macular vision of one of them, and hence must be concluded to be due to variations in the amount of the macular pigment.

This conclusion may be still further supported from the following considerations: Sachs[18] measured the spectrophotometric transmissions of freshly prepared specimens of the human retina from nine individuals. König[19] calculated mean figures for the transmission of the macula from Sachs' data. Ives[20] recently outlined a method of determining the dominant hue of a selective substance, given its coefficients of selectivity and König's "sensation" curves. Following this procedure, the dominant hue of the macular pigment with König's sunlight as the illuminant turned out λ579.5 mμ, in almost unwarranted agreement with λ578 as the intersection-point of the curves of Fig. 5. Dominant hue is of course a function of the spectral distribution of the illuminant, so that it is not to be expected that the value given for the macular pigment by König's data would coincide exactly with that given by the results of the present investigation. The fact that the two agree as nearly as they do leaves no doubt that in both cases the same pigment has played the role of the variable factor.

There is, however, an occasional individual who altho in no degree color-blind is clearly anomalous. One such was encountered in the present investigation in the person of observer (AE). His individual observations show no greater or at any rate not significantly

greater probable error than those of the majority of the observers, which is proof that he is not color-defective. The amount by which his white-point deviates from the line of normal variation is in terms of hue-difference relatively very considerable. It is again to be concluded, however, that the principal factor in the variation is a selective modification of the radiant energy reaching the retina, perhaps due to the presence of some unusual coloring matter in the media of the eye rather than to variation in amount of some pigment already present in the normal eye, since the other observers show no tendency toward a two-dimensional variability. The variation of (AE) is toward the red from the normal, which fact suggested the possible tinting of the ocular media by the bursting of a blood-vessel on the retina but that hypothesis appears to be ruled out by the circumstance that the observer's color-vision seems to be alike in both eyes, although this point was not carefully enough tested to make a positive statement.

Cases of "anomalous" three-dimensional or "trichromatic" color-perception have been reported in the literature. König (17) obtained complementary data from two observers whom he styles "anomalous". These data are shown plotted in Fig. 5 and determine the dotted curves B and Z. In the original figure from which Fig. 5 is copied only the lower portions of these curves are drawn. In the accompanying table in König's paper a few additional points are given, however, which have been used to fill in the rest of the curves. The white illuminant was in this case "gas-light".

For the sake of comparison, König obtained also sets of

complementary data for himself and D_ieterici with "gaslight" as the white illuminant, and these determine the dotted curves K and D in Fig. 5. König points to the evident grouping of the curves into distinct sets as proof that the former observers are "anomalous". Note, however, that when their curves are extended they are found to bear about the same relation to one another as those of König and Dieterici, including an intersection in a out the same relative location, thus the "anomalous" observers closely resemble one another and their mutual differences are apparently of the same type as those of the normal observers K and D. Both observers E and B were also subjected by König to a complete three-color analysis. The data are given in full in the same treatise to which reference has been made. A comparison of these data with the similar data of K and D shows no such discrepancies. The "Elementarempfindung" curves of E have been plotted by König together with those of K and D, from which it is evident that the complements of the extreme hues of the spectrum, given by the points of intersection of the curves, differ less for D and E than for D and K and what difference exists is in the opposite sense from that shown by the curves reproduced in Fig. 5.

Apparently no actual abnormality existed in these cases. More probably, the differences exhibited in Fig. 5 are due to a variation in the quality of the "gas-light" illuminant. Altho there is no definite statement to that effect, it seems probable from the context that the two pairs of observers were examined on

separate dates. A description of the gas-burner is also wanting. The color of a gas flame is known to vary considerably with a difference in the burning conditions, or in the composition of the gas. Konig's specific assertions as to the abnormality of these observers are not supported by his data.

As to the cases of "anomalous" color-perception which have been reported by other investigators no definite conclusions can be drawn, because the tests applied to them have concerned merely the form of the "visibility" function of the individual, which need not necessarily bear any relation to his color-characteristics. It is to be noted that among the fourteen observers whose data is plotted in Fig. 4 certainly no marked abnormality occurs except in the single case which has been discussed. The standard white variations superposed upon the characteristic ones prevent an absolute statement.

Besides the major variations affecting the location of the white point, Fig. 4 shows in most cases a failure of the groups of complementary lines to intersect precisely in a point. One notices, however, that the configurations of the separate sets bear a certain resemblance. In particular this is true of the portions of the figures formed by the lines joining the blue-yellow and the violet-green complementary pairs, which display a divergence toward the blue corner of the diagram, that is more marked in the figures lying nearer the blue. This is true without exception, even of the configurations formed by König's own data at K and K'.

Apparent exceptions are the figures of [illegible] and T, but these investigators did not extend their complimentary observations into the extreme violet, so that the complementary pairs represented in the figures are not the same as in the other cases. In other cases complimentary pairs corresponding approximately to those selected for the present investigation were plotted so as to have the results in the most comparable form.

Thus it is evident that an alteration in the color-triangle in the region of the blue or violet would materially reduce the total area included by the small figures, hence one may conclude that the triangle is in error in this region.

On the other hand as regards the pairs of lines ending in the blue-green, which cut the former pairs more or less transversely, the apparent discrepancies are greatly magnified by the distortion of the diagram approximately at right angles to them. In no case would a change in the blue-green member of a complementary pair greater than one millicron be needed to cause the corresponding line to pass through the respective white-point marked in the diagram. Taking into account possible small errors in the color-triangle in the blue-green or red regions together with the probable observational error of the complementary determinations, which is greatest of all for the pair including the extreme red, the failures to intersect are certainly within the limit of error of the investigation.

The configuration enclosing the greatest area is that of

(PFG) whose data as previously stated also have the greatest probable error in the present investigation. It may be interjected at this point that three prospective observers were rejected in preliminary investigations on the grounds that the deviations of their individual settings were unduly great. There was no reason to suspect that their color-characteristics differed from the rest. Apparently the causes of the greater uncertainty lay elsewhere than in a defective color-sense. In one case at least the trouble seemed to be in refractive defects, since the individual was unable to distinctly see the dividing line between the halves of the photometric field.

The conclusion has therefore been reached that the results of the present and previous investigations fail to reveal inherent variations in the complementary wave-lengths of separate individuals other than can be accounted for by differences in the absorptive properties of the ocular media, and that if variations of other types occur, they are so small as to be barely detectable through the greatest attainable accuracy in color-matching. While there is an expanse in the green not covered by the complementary statistics it seems safe to infer that the generalization can be extended to all color-match equations.

The situation seems sufficiently general to warrant the use of the complementary data as criteria for testing the accuracy of the various extant color-triangles. A few representative sets were plotted on Maxwell's two triangles and that of Dieterici as

well as that constructed from the average data of Konig, Dieterici and Abney by Weaver (7). Space does not permit publishing the diagrams. The trianlges of Konig and of Maxwell's observer (J) are about equally good, and are somewhat better than the rest, including the average triangle, according to this standard.

The data of the present investigation determine a mean white-point for Konig's triangle at about the location of the white-point of observer(RHS), which is also the median point. The original white-point of the triangle, shown in Fig. 4, happens to lie very near here.

Relative to a possible relationship between the amount of pigment present on the macula and on the iris, it was noticed that the only blue-eyed observers (HTW) and (AE), show the greatest amount of absorption in the transparent media; but the relation, if any exists, is evidently not close because some of the observers whose points lie near the middle of the range have very dark eyes, while the iris of the observer with least macular pigment is light brown or "hazel".

The question now suggests itself as to whether the individual "visibility" function or <u>luminous efficiency</u> function (to adopt the more descriptive terminology of Ives) is invariant if correction is made for the absorption of the ocular media. The question is not answered by the present investigation as satisfactorily as is to be wished; apparently, however, the case is otherwise.

The present and other experimental investigations show that

luminosities are distributed on the plane of the color-triangle very nearly according to the same law that applies to hue. Therefore to every point in the color-plane in addition to the two coordinates describing its hue can be attached a third coordinate proportional to its luminosity thus completely fixing its position in the color-space. The knowledge of the three coordinates of three points is sufficient to determine the plane, and therefore to fix the luminosity coordinates of all points on it. That fact has already been used in finding the luminosity ratios of complementaries from the color triangles. The assumption of an invarient luminous efficiency function is equivalent to assuming that the color-plane has identical luminosity coordinates for all individuals.

Turning to Fig. 4, it appears that the pair of spectral hues given by the intersections of the line ab with the spectral locus are complementary for all the observers whose white-points lie along it. Supposing the luminosities of these spectral hues to be invarient on the color-plane, mixtures of them which are complementary to the different observers would have luminosity ratios dependent in a definite manner upon the locations of the different observer's white-points. The luminosity ratio of longer to shorter wave-length would be greater corresponding to white-points lying nearer the longer wave-length hue. The exact relation in which the luminosity ratio should stand in the different cases can be worked out from the diagram.

The universal pair of complementaries was not one for which the luminosity ratios were experimentally determined.

However, the pair including the terminal violet is practically invarient, showing perceptible variation only in the extreme case of (HME). The luminosity ratios of this pair given in Table III do not exhibit variations of the sort anticipated from the initial postulate. The ratio of (HTW) which on the above hypothesis should be greatest, is actually least and is less than half that of one of the other observers, while that of (HME) which should be least, is next to the highest.

These differences are certainly larger than the probable errors given by the deviations of independent settings. It is possible, however, that the results are influenced by the Purkinje effect more in the case of some observers than of others. A general discussion of the Purkinje effect in connection with the observations was entered into, in the preceeding section. It has been suggested that structural differences of the retina may exist which would render some individuals liable to the Purkinje effect on the foval area, while others are free from it.(1) From this circumstance, the evidence of independent variation of the luminous efficiency function is regarded as inconclusive.

Supposing that such variations do occur, it may be pointed out that the state of affairs is similar to what might be expected if three distinct photochemical substances were present in the receptors of the retina. Since color-equations when corrected for absorption in the ocular media are found to hold universally as far as present experimental evidence is capable

of showing, the most general transformation of the color-variables x,y,z from one normal individual system to another has the linear homogeneous form,

$$x_2 = a_1x_1 + b_1y_1 + c_1z_1$$

$$y_2 = a_2x_1 + b_2y_1 + c_2z_1$$

$$z_2 = a_3x_1 + b_3y_1 + c_3z_1$$

where a,b,c are all positive.

The simplest form of the transformation compatible with invarient color-characteristics but varying luminosity-perception is

$$x_2 = ax_1$$

$$y_2 = by_1$$

$$z_2 = cz_1$$

which would be realized in the case of three photo-chemical substances possessing definite response functions $x(\lambda)$, $y(\lambda)$, $z(\lambda)$, present in different relative amounts in different individuals.

In conclusion the writer wishes to make acknowledgment to the members of the Physics class who kindly furnished observations, and to Dr. Ifund for several most useful suggestions.

Table I.

λ	λ-λ	$\frac{L_\lambda}{L_w}$	$r_1 - r_2 - r_w$
Extreme Red	496.5	.75	2.2
609	493.5	1.10	3.8
591	490	2.07	3.5
586	487.5	2.84	2.2
580	482.5	4.8	2.4
578.5	480.5	5.9	1.9
576.5	474.5	9.2	- .2
574	472	11.6	1.2
573	466.5	16.0	1.4
572	459	24.5	1.7
570.5	xtreme Violet	40	1.7

Observer [illegible]

Table II.

	λ_1 (given)				
	Red	609.5 mμ	581	Violet	587
Observer	λ_2 (observed)				
A H P	498 mμ	494	487	569.5	
A E	500	495		582	479.5
R M	495.5	492.5	484.5	569	
H C W	503.5	500.5	491	569	
P F	494	491	485*	569	
H E L	489.5	487	480	566	
E F G	495*	492*	487.5*	566.5	
	Na Cr O_3 filter introduced.				
R H S	501	498.5	487.5	571.	
	Color-temperature of source increased. C T = 5000+.				
E H S	495	492	483	569.5	
	Sunlight filter removed. C T = 3000°.				
R H C	502	500	479	575.5	
		$\Delta \lambda_1$			
		1	3		
		Mean $\Delta \lambda_2$			
	16	16	16	2	

Starred observations excepted, all probable errors not greater than 1 mμ.

Table III.

Given W.

Observer	Red	609.5 mμ	581 L_1/L_2	Violet	L_1 L_2 - L_ω (average)
A N P	.58	.96	3.3	26	4.2
[illegible] E	.28	1.08		20.5	5
H [illegible]	.64	.85	1.85	17.5	10.5
[illegible] H [illegible]	.67	1.0	3.8	36	6.1
H [illegible]	.55			28.5	4.1
Konig's triangle	.36	.72	3. 8	5. 8	
Average triangle	.51	.96	6.4	82.5	
Helmholtz	.44	1.00	4.0	10.0	

Legend.

v. K von Vries

v. F. von Frey

H. Helmholtz

K. König

D. Dieterici

A. Angier

T. Tremelenberg

K' König ' extra-macular,

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Biographical Note.

Roger Hull Sinden, son of George and Mabel (Hull) Sinden, was born at Oak Park, Illinois, in 1895. He received a High School education at Canon City, Colorado. In 1918 he was awarded the degree of Bachelor of Arts by Colorado College. He served in the Signal Corps branch of the military service of the United States from September 1918 to May 1919. The following academic year was spent pursuing graduate study in Physics at Dartmouth College. In 1920 he received the degree of Master of Arts from Dartmouth College. He was employed at the Laboratory of Applied Science of the National Lamp Works of the General Electric Company. in the capacity of assistant physicist from August 1920 to January 1922, when he resigned in order to commence, in February 1922, graduate study in Physics at the Johns Hopkins University. He followed the courses of Professors Ames and Wood in Physics, and Cohen and Murnaghan in Mathematics.

His publications are as follows:

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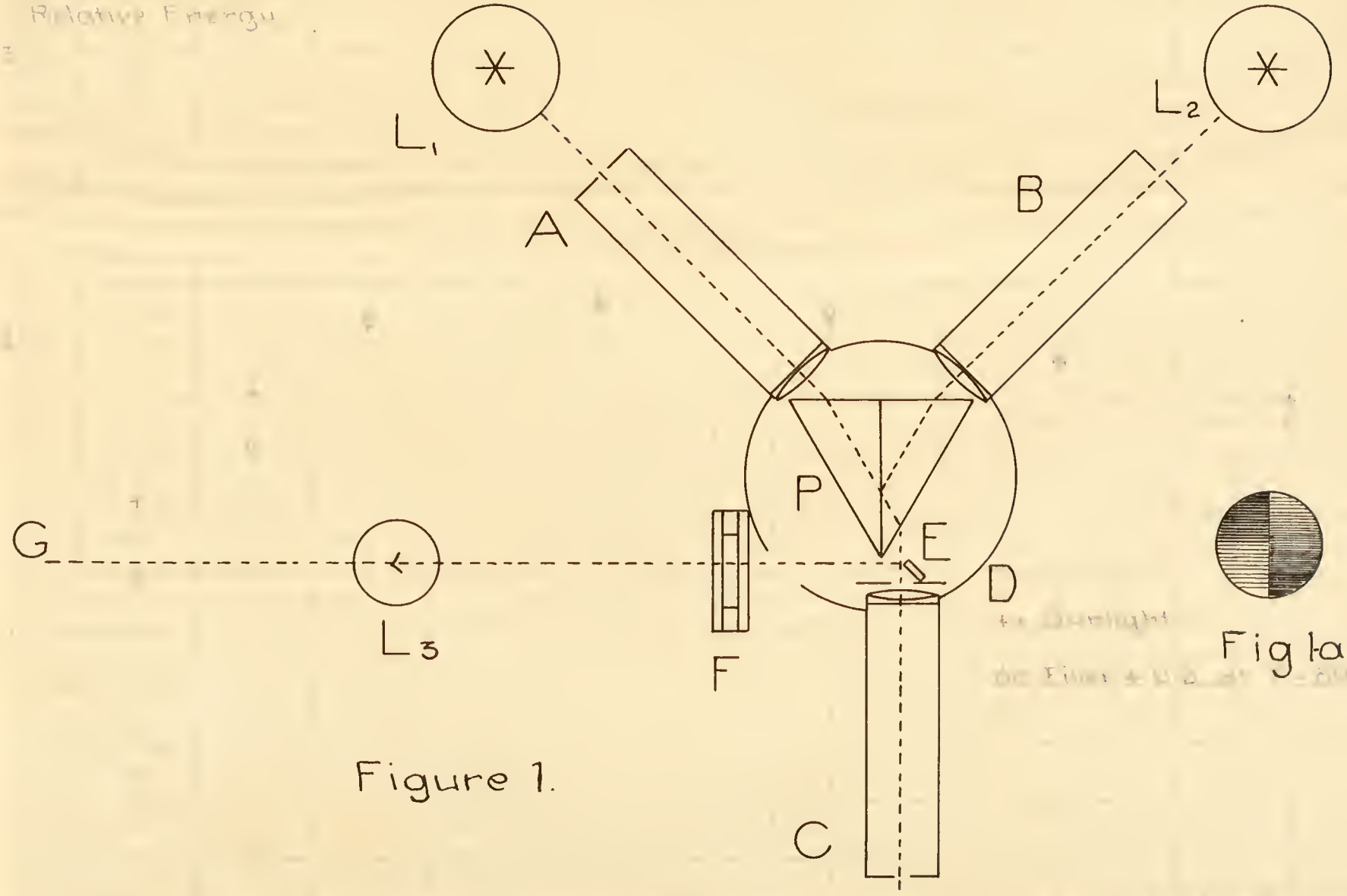


Figure 1.

Fig 1a.

Figura 1
Figura 2
A
B
C
D
E
F
b
C'
r1
r2
r3

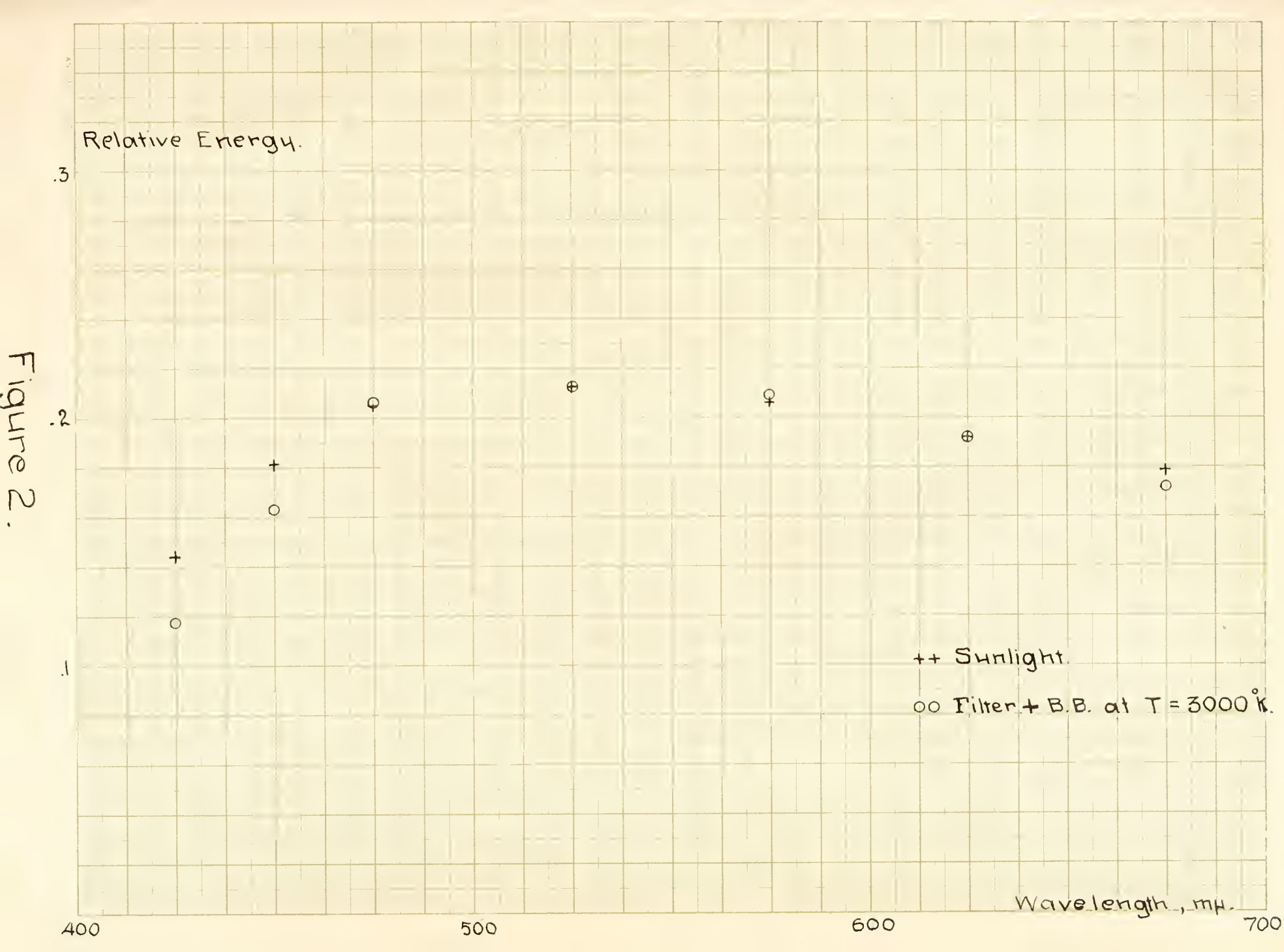
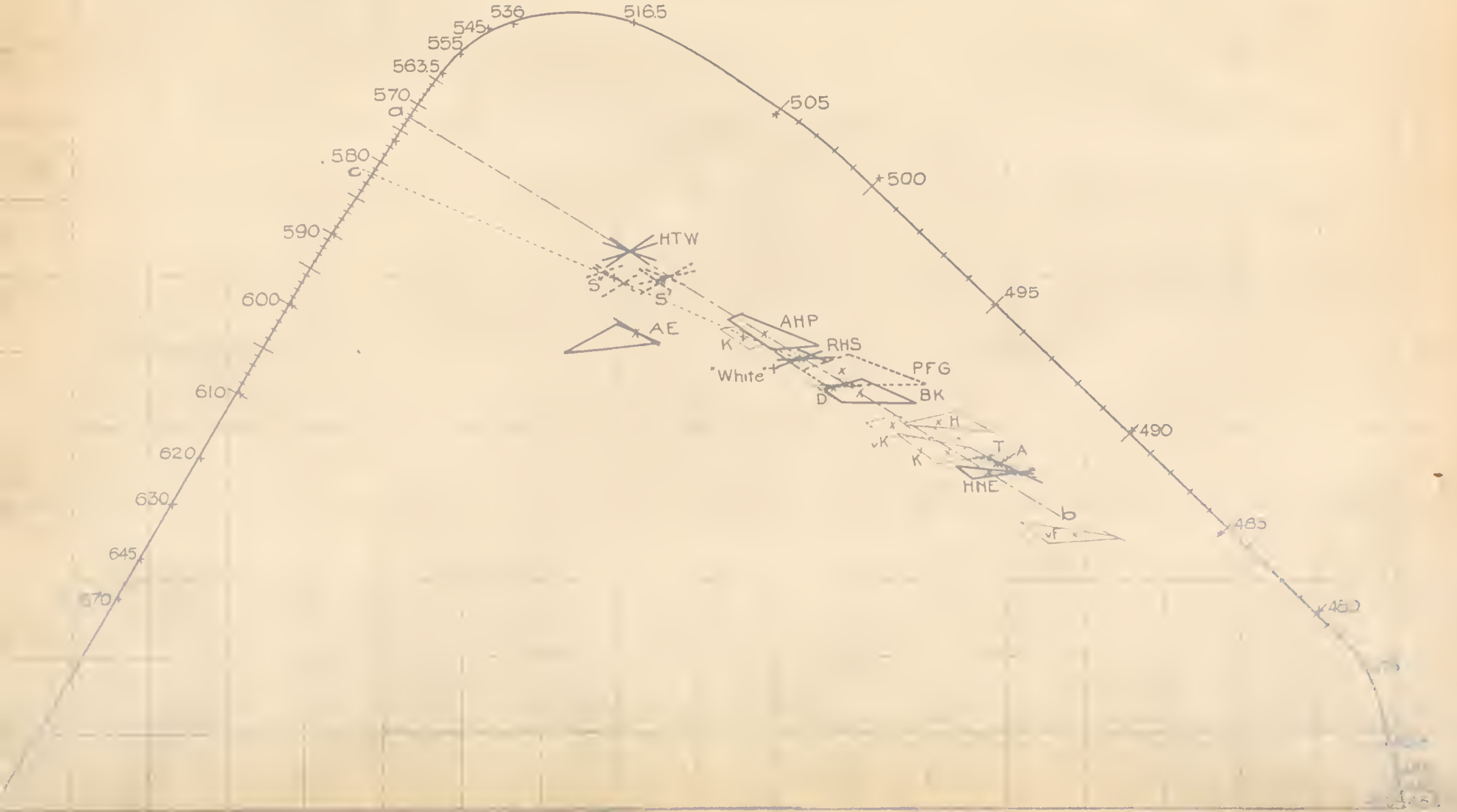

Relative Energy.
.3
.2
.1
++ Sunlight.
oo Filter + B.B. at T = 3000°K.
Wavelength, mμ.
400
500
600
700

Figure 2.

40
30
20
10
0
Relative Subjective Saturation.
400
500
600
700
Wavelength mμ.

Figure 3.


536
516.5
545
555
563.5
570
a
580
c
590
600
610
620
630
645
670
505
500
495
490
485
480
HTW
S
S
AE
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K
RHS
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K
T
A
HNE
b
F

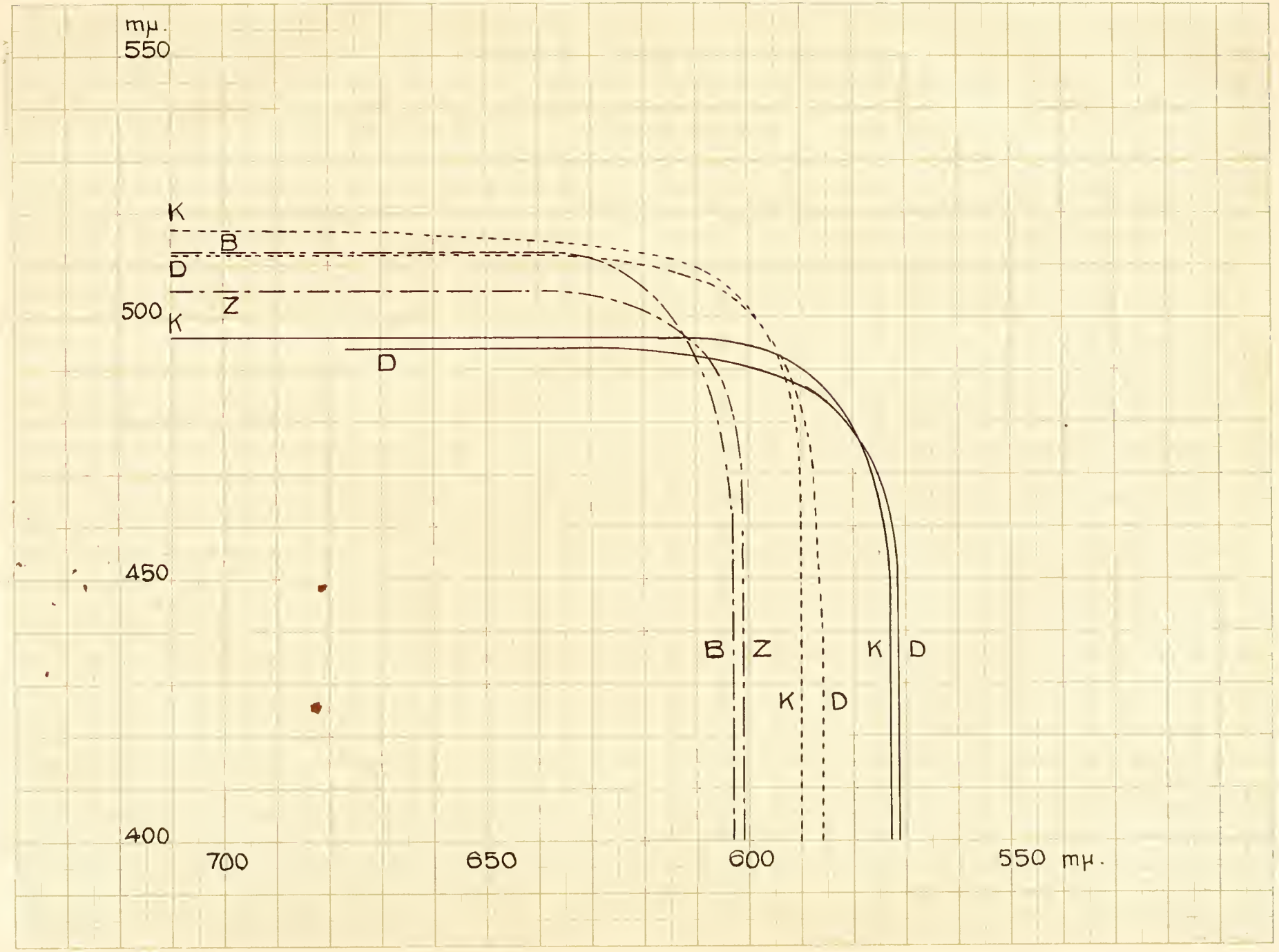

Figure 5.